Singularity Is All You Need?

AI, Structural Unemployment and the Baumol Inversion in a Pure-Labour Economy

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Abstract

For most of the last two centuries, sectors with technological breakthroughs and fast productivity growth offloaded labour, which was re-employed in sectors these innovations left largely unperturbed and where demand kept growing. Fordism made the factories more efficient, but left schooling largely untouched. This negative correlation between productivity growth and demand growth is what kept the released labour employed. The promise, or fear, of AI is to invert it; the promise of AGI is machines that outmatch us at every task. I follow that promise through Pasinetti's pure-labour structural dynamics. One condition, that the labour required per head, summed across sectors, equals one, is at once the condition for full employment and for the full spending of income. If AI raises productivity growth above demand growth in every sector, the labour requirement declines, and no reallocation of demand can restore it. The economy does not collapse; it settles into an underemployment equilibrium in which the surplus of those in work supports those out of it, and employment depends on the labour content of what that support buys. I also discuss the alternatives: a floor on how far automation can go, shorter hours and new sectors. The framework separates what the technology permits from what institutions do with it, and I use it to read four public claims about AI: a world without money, the end of the economic problem, gains that do not show up in national income, and a universal basic income.

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Competing interest. The author runs ClicheFactory, which sells AI software that automates document work.
Use of generative AI. The author used Claude Code (Anthropic) throughout the work: to discuss and test the argument, to write first drafts and edit prose, to typeset the equations and format the paper, to write symbolic (sympy) checks of the derivations, and to search for references. The author edited every paragraph and is responsible for the content of the paper.

1 Introduction

For two centuries, industrial economies have been able to re-employ the labour that technology released in the sectors that technology changed least. First agriculture and then manufacturing needed ever fewer

workers per unit of output, and as incomes rose, demand moved towards services, where productivity grew slowly and demand for labour was strong. Baumol (1967) called the rising relative cost of those services a disease, but the disease was what kept people in work. The productivity effect of AI seems poised to land hardest in precisely those sectors. If this were to happen, then the sink that absorbed displaced labour would be gone, and with it the reassurance that new work always turns up.

Since OpenAI turned the findings of *Attention Is All You Need* (Vaswani et al. 2017) into a product, AI has had much of ours. It has changed how we interact with technology, it has become the object of a global race for technological dominance, and markets are pricing AI companies with the bipolar frenzy typical of bubbles. Its future is likewise Janus-faced, and rarely without skin in the game: doom on one side, sometimes suspected of being argued to fence off open-source competition, and on the other the *Machines of Loving Grace* of Amodei (2024) and, further out, a Star Trek economy in which scarcity, and with it money, disappears, a vision that makes strange bedfellows of Yanis Varoufakis and Elon Musk. None of these futures is likely to arrive as its seers imagine it, but the transformation is real.

Economists have traditionally met such technological claims with a calm belief that our apparatus has seen many revolutions and managed to absorb all of them: productivity shifts, the economy finds a new steady state, and the gains reach society at large with a lag. That is a caricature of the literature but a fair summary of where policy debate eventually ends up, and it is the attitude that led Krugman (1998) to predict that the internet would have no more impact on the economy than the fax machine. There are two ways to misjudge a technology: to call every fad a paradigm shift, or to trust an apparatus so general that every change can be absorbed by it, which assumes the problem away. Ricardo avoided the second: in "On Machinery" (Ricardo 1821, ch. 31) he concluded that machinery can (but need not) injure the labouring class. So the question is worth asking: what if AI is a genuine structural break, neither the end of the world nor the end of scarcity, but a change that reshapes our societies all the same?

This paper does not forecast which future arrives. It asks what each would require, including some less savoury adaptations to yet another Promethean gift. Public debate around AI runs two questions together: what the technology makes possible, and what a society will do with it. Detractors and proponents will hear what they want. "AI can do this task" is heard as "the job will disappear", and "AI will make everything cheap" is interpreted as "an age of abundance for all". The first half of each claim is about technique, the second about institutions, and the disagreement is mostly about the second. The framework used here keeps the two apart.

I use the pure-labour model of structural change developed by Pasinetti (Pasinetti 1981, 1993). Its advantage is that a short list of assumptions yields a long list of results, all from one condition: that the labour required per head of population, summed across sectors, equals one (B5). Two results matter most for this debate. The first is the inversion: if AI raises productivity growth above demand growth everywhere, and most where demand grows fastest, the labour requirement falls, and no reallocation of demand across existing goods can restore it (Proposition P4). The second is what the system settles into when the condition fails. It does not collapse. It reaches an equilibrium in which part of the population is supported outside employment, and in which the employment effect of that support depends on who produces what it buys (Proposition P12, Proposition P14). Section 8 then reads four public arguments through the same condition: that AI will produce a world without money, that it will solve what Keynes (1930) called the economic problem, that its gains will not show up in national income, and that a universal

basic income will settle the matter.

Section 2 explains the choice of framework and Section 3 sets it out. Section 4 to Section 7 derive the results, and Section 8 addresses the four arguments.

2 Why this framework

Three features of the question decide the framework: AI is a form of learning, it changes some sectors far faster than others (software, for example), and the worry is what that does to employment. Pasinetti's structural dynamics is built around all three. It starts from the observation that changes in the level of macroeconomic magnitudes always come with changes in their composition (Pasinetti 1993, 8), treats technical change, his "social learning", as a permanent feature of industrial economies (Pasinetti 1993, 36), and measures the whole system against full employment. So the question becomes: in a society of constant technological progress, with mass AI adoption, will there still be enough work? The answer depends on where AI raises productivity relative to where demand grows, so the framework has to let both vary from sector to sector, and independently of each other.

2.1 A structural apparatus with technological change at its core

This paper is concerned with uneven productivity growth across sectors and the ensuing reallocation of labour. Each sector has its own rate of per-capita demand growth r_i in (A3) and its own rate of productivity growth ρ_i in (A2); Section 3 defines both. There is no assumption about these rates being equal across sectors: a common rate is a special case that is neither realistic nor interesting, since there are no structural dynamics to speak of. Quite often the modelling process begins with a one good economy, corn is often used as an example, where productivity is a scalar and technical change is a shift of a single frontier. Sectoral asymmetry can be reintroduced there, but only as a complication. This investigation is concerned with how non-uniform, AI-induced changes in productivity growth across sectors impact the economic system, and there is no reason for this to be an afterthought instead of being at the core of the analysis. So I use a framework that has the asymmetry as its primitive and does not have to be bent to incorporate it (Pasinetti 1981, 1993).

There exists a task-based literature on technological change and employment (Acemoglu and Restrepo 2018, 2019). Production is decomposed into tasks, labour gets displaced by automation and this allows for a creation of new tasks where labour has the comparative advantage. The canonical form of the tasks framework does not have a demand side income-elasticity structure. Output is a single final good produced from various tasks. If we admit additional industries, the composition effect of Acemoglu and Restrepo (2019), would mean reallocation is driven by relative prices (themselves based on the technology of the tasks) and not by changing demand for different commodities as per capita income increases. Displacement is a property of technology; the destination of displaced labour is not modelled as a property of demand. This is where the Baumol inversion comes in (Section 4): the historical sink of Section 1 was as much a property of demand as of technology. A framework without a demand side is blind to that, so it cannot tell us what happens when displacement comes for that very sink.

2.2 Demand as data

Differences in demand growth will be shown to be just as important as differences in productivity growth. Consumption decisions are not modelled, per-capita demand for each good simply grows at its own rate, with no assumption that the rates are proportional to one another or to income. The rates are data: the model takes them as given, and nothing below depends on why they take the values they do, only on the fact that they differ. That is what lets a change in the *level* of income bring a change in its *composition*. With uniform consumption, composition would reflect the moves in relative prices, set by the production system, and the model would be blind to demand's effect on the structure of output and employment.

Furthermore, Pasinetti argued that a dynamic analysis cannot be agnostic about the evolution of demand in a situation where productivity and per-capita income are continuously increasing. He takes as a stylized fact that "the structure of consumption always varies as income changes", and the best-known instance is Engel's Law, that as real wages rise the proportion of them spent on food falls (Engel 1857; Pasinetti 1993, 38). While not a law, it is nevertheless a reasonable description of consumption dynamics: in spite of rising incomes there is a limit to our soda and sandwich consumption, while other categories, such as leisure, healthcare or education, might, also up to a point, fill the gap. There is a silent assumption when discussing capitalism, I think, that there will always be a new sector that will fill this gap. As for this investigation, Engel's law is a reasonable working assumption but the arguments of this paper do not rest on it; all the model needs is that the r_i differ across goods.

Two things are lost without demand effects. First, the stabiliser Proposition P3, and with it the sectoral migration that history shows, from agriculture and manufacturing to services. The stagnant sectors kept absorbing labour because demand for their output kept growing in spite of the rising relative price of their product, that is, because r_i was high precisely where ρ_i was low. Without an independent $\{r_i\}$ nothing produces that pairing. Second, demand and technique move in unison, and the question stated at the top of this section can only be answered at a knife-edge, $\rho_i = r_i$ for all i, an empirical improbability and therefore practical irrelevance. The question is about the joint distribution of $\{\rho_i\}$ and $\{r_i\}$, and without an independent $\{r_i\}$ it cannot be posed.

2.3 Technology and institutions

Pasinetti builds his investigation in two stages (Pasinetti 1981, 1993). The first stage models a natural system of prices, a natural rate of interest and the macroeconomic condition under which income is fully spent and labour fully employed. This stage doesn't say who owns what or how income is distributed and how markets clear. The second stage discusses the necessary institutional conditions that bring an actual economy onto that path. Public discussion concerning AI runs the two stages together (Section 1); keeping them apart is what the natural system is for.

(B5) belongs to the first stage: it says what the path of per-capita demand would have to be, given the path of labour coefficients, for everyone who wants to work to be employed. There is no assumption that says it will be, and as Proposition P1p makes clear, the model doesn't enforce it. When the condition fails, what happens next, whether shorter hours (Proposition P10), new sectors (Proposition P11) or unemployment, with part of the population supported outside employment (Proposition P12), is an institutional question. The natural system is therefore a normative benchmark rather than a forecast: a

forecast of how a market economy will respond to AI would need a theory of how firms, households and governments behave under a quickly evolving technology, whereas a benchmark needs only the technology and the composition of demand. It measures the gap between what the technology permits and what the institutions deliver, and that gap is the subject of the second half of the paper.

2.4 Assumptions and limitations

Three limitations should be mentioned up front. There is no choice of technique: the ρ_i are exogenous, AI enters as a change in these rates and not as a modeled decision. The same exogeneity holds for demand: the r_i are data, set independently of the growth of income, and in Section 6 that independence has a cost. Labour is homogeneous, so the model can only say how much labour is displaced but not what kind.

3 The model

In Section 2 I ask whether or not, after large scale AI adoption, there will still be "enough work". This section provides the natural stage of the analysis (the benchmark). Nothing here is explicitly about AI yet.

3.1 Notation

Symbol	Meaning
t	continuous time
$N(t)$	population, equal to the labour force; $N(t) = N(0)e^{gt}$
g	rate of population growth
$i = 1, \ldots, k$	consumption goods
$a_{ni}(t)$	labour coefficient of good i (labour per unit output)
ρ_i	rate of labour-productivity growth in sector i
$c_i(t)$	per-capita demand for good i
r_i	rate of growth of per-capita demand for good i
$X_i(t)$	total physical output of good i
$L_i(t)$	labour employed in sector i
$\varepsilon_i(t)$	L_i/N, labour requirement per head of population in sector i
$E(t)$	$\sum_i \varepsilon_i(t)$ — total labour requirement per head.
$e(t)$	employment rate
$\mu_i(t)$	expenditure share of good i
$p_i(t)$	natural price of good i
$w(t)$	wage rate
$h(t)$	labour supplied per head (working day/week), normalised to 1 in the baseline
$\bar{a}_i$	human-in-the-loop floor on a_{ni}

Symbol	Meaning
$\hat{\rho}_i(t)$	measured productivity growth in sector i under a floor; tends to zero
$\rho^*(t)$	natural rate of interest; growth rate of real per-capita income
A^e	labour absorbed by the employed basket
A^u	labour absorbed by the non-employed basket
A^k	labour absorbed by the elite basket
a_{in}	demand for good i per unit of labour — the mirror of a_{ni}; equal to c_i while each head supplies one unit, c_i/h in general
A	the coefficient matrix of the closed physical system, $A \cdot X = 0$
θ	within-phase time, $\theta = t - \tau z$; the clock resets at each phase boundary
z	length of a phase
τ	phase index
a_{Kj}	capital coefficient — capital good K (price p_K) per unit of good j
δ	rate of depreciation
π_j	rate of profit in sector j

3.2 Primitives

$$N(t) = N(0)\, e^{gt} \tag{A1}$$

$$a_{ni}(t) = a_{ni}(0)\, e^{-\rho_i t} \tag{A2}$$

$$c_i(t) = c_i(0)\, e^{r_i t} \tag{A3}$$

The model rests on three exogenous laws. Population, which is also the labour force (everyone in it can work, so there are no dependants), grows at rate g (A1). The labour needed per unit of good i falls at a sector-specific rate ρ_i (A2),[1] and per-capita demand for good i grows at a good-specific rate r_i (A3). I have argued in the previous section why they are taken as given. Every derivation below follows from (A1)–(A3) plus accounting. AI naturally enters as a change in $\{\rho_i\}$, without changing the original framework.

[1]Three symbols differ from Pasinetti (1993): his l_i is a_{ni} here, as in Pasinetti (1981); his unnamed product $c_i l_i$ is ε_i; and per-capita demand is c_i, written a_{in} only in (B7). The subscript n always means labour, never a growth rate.

3.3 Quantities and the condition

$$X_i(t) = c_i(t)\, N(t) = X_i(0)\, e^{(r_i+g)t} \tag{B1}$$

$$L_i(t) = a_{ni}(t)\, X_i(t) \tag{B2}$$

$$\varepsilon_i(t) = \frac{L_i(t)}{N(t)} = a_{ni}(t)\, c_i(t) = \varepsilon_i(0)\, e^{(r_i-\rho_i)t} \tag{B3}$$

$$E(t) \;\equiv\; \sum_i a_{ni}(t)\, c_i(t) \tag{B4}$$

$$E(t) = \sum_i a_{ni}(t)\, c_i(t) = 1 \tag{B5}$$

$$e(t) = \min\{E(t),\, 1\} \tag{B6}$$

Output is per-capita demand times the population (B1), and employment in a given sector is its labour coefficient times output (B2). Dividing L_i by the population gives the labour requirement per head, ε_i (B3). Its growth is determined by $r_i - \rho_i$, falling where productivity outpaces demand and growing where demand outpaces productivity.

Summing over sectors gives $E(t)$, the total labour requirement per head (B4). E is labour demanded, not labour supplied: if $E = 3/4$, the economy needs three quarters of the labour its population can provide. Whether a quarter of the population is unemployed or everybody is working three quarters of the time is not determined by the model. (B6) assumes the first, and Section 6 takes up the second. This is the number that the previous section asks about: when E equals the labour each person supplies, normalised to one, the system provides enough work. The condition does not have to hold, and most of the discussion below is about the possible paths of E.

When the condition fails, E can be below one, in which case every sector produces to satisfy demand and there is excess unemployment, with the employment rate being E (B6). Above one, demand is outpacing supply and cannot be met in full. The latter is handled in Section 6. The former is explained in Section 7.

One caveat applies throughout: comparing labour requirements presumes the same basket of goods, and where AI changes the composition (a realistic assumption) and not just the efficiency, a change in E mixes the two. For simplicity, AI changes only how goods are produced: not which goods exist, and not how demand for them grows. The set of goods changes only when one leaves the commodity space (Section 5) or a new one enters (Section 6). The entry of new goods is exogenous to the model and could be an interesting avenue of future investigation.

3.4 The condition as a (different kind of) singularity

(B5) can be stated a second way. The construction is Pasinetti's (Pasinetti 1993). Writing demand per unit of labour as a_{in}, the mirror of a_{ni} (it is c_i, since each head supplies one unit), and treating labour as the $(k+1)$th commodity, the quantity system is homogeneous:

$$\begin{aligned} X_i - a_{in} X_n &= 0, \qquad i = 1, \ldots, k \\ X_n - \sum_i a_{ni} X_i &= 0 \end{aligned} \tag{B7a}$$

that is, $A \cdot X = 0$ with

$$A = \begin{bmatrix} 1 & 0 & \cdots & 0 & -a_{1n} \\ 0 & 1 & \cdots & 0 & -a_{2n} \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & 1 & -a_{kn} \\ -a_{n1} & -a_{n2} & \cdots & -a_{nk} & 1 \end{bmatrix} \tag{B7}$$

A non-trivial solution requires $\det A = 0$, and by the Schur complement on the bordered identity

$$\det A = 1 - \sum_i a_{ni} a_{in} \tag{B8}$$

so $\det A = 0$ if and only if (B5) holds: the employment condition *is* the singularity condition of the closed system.[2] The price system $p_i = a_{ni} w$, $w = \sum_i a_{in} p_i$ is

$$A^\top p = 0 \tag{B9}$$

and since $\det A^\top = \det A$, it is solvable under the same condition.

These are two separate systems: neither is solved by the other. They share only their condition of solvability. The condition leaves one degree of freedom in each system, and each is closed from outside: the quantity system by the size of the labour force, the price system by the wage or a numéraire. This is a pure-labour form of openness of distribution found in Sraffa (1960) with the wage rate as a given.

3.5 Value, prices, income

$$p_i(t) = a_{ni}(t)\, w(t) \tag{C1}$$

$$\sum_i p_i(t)\, c_i(t) = w(t) \tag{C2}$$

[2]The model's singularity, a vanishing determinant, is the benchmark against which I read the consequences of the technologists' singularity, in which AGI comes to dominate every domain.

$$\mu_i(t) = \frac{p_i c_i}{\sum_j p_j c_j} = \frac{a_{ni} c_i}{E(t)} \tag{C3}$$

When (B5) holds,

$$\mu_i(t) = \varepsilon_i(t) \tag{C4}$$

$$\rho^*(t) = \sum_i \mu_i(t)\, \rho_i \tag{C5}$$

$$\frac{w}{p_i} = \frac{1}{a_{ni}} \tag{C6}$$

(C1) is the labour theory of value in its pure form, with the natural price of each good being determined by its labour content at the wage. For readers more accustomed to utilitarian calculus, think of it as a simple world of constant disutility. Prices only exist for goods with labour input, making $a_{ni} > 0$ a condition of the value system. It holds in everything up to Section 5, which takes up what happens when AI pushes a coefficient to zero.

The expenditure condition (C2) says that per capita spending has to equal the wage if we want full employment income to be fully spent. Expenditure shares are ε_i/E, with the wage cancelling (C3): each is the sector's share of those in work, at any level of employment, and where (B5) holds, also its share of the labour force (C4).

We get ρ^* (C5), the growth rate of the real wage (and of real income per head at full employment) and the natural rate of interest of the pure-labour system by weighting sectoral productivity growth by expenditure shares (Pasinetti 1993). The real wage in terms of good i is $1/a_{ni}$ (C6) and grows at ρ_i. ρ^* weights sectors by spending, so a sector's contribution to growth depends on how much is spent on it, a point Section 5 turns into a result.

3.6 The identity and the condition are not the same thing

Proposition P1. *The expenditure condition and the employment condition are the same*

> Substituting (C1) into (C2) gives $\sum_i a_{ni} w c_i = w$, hence $\sum_i a_{ni} c_i = 1$, and therefore (C4): expenditure shares equal shares of the labour force. Equivalently by (B7)–(B9): the quantity system is $AX = 0$ and the price system is $A^\top p = 0$; the two are solved independently of each other. However, a square matrix and its transpose share a determinant, so both admit non-trivial solutions under the single condition $\det A = 1 - \sum_i a_{ni} a_{in} = 0$.

Verification of the coincidence comes from the first proof and the second one explains it. Reading from the quantity system it states that available labour is fully employed, read through the price system, the

national income is fully spent. One equation with value from Smith (1776) and effective demand from Keynes (1936). Demand failure and technological unemployment are two sides of the same coin.

Proposition P1p. *Why the condition is a condition and not a tautology*

(A2) and (A3) are two exogenous dynamic laws: the path of a_{ni} is a fact about technology, the path of c_i a fact about preferences at full-employment income. Neither is chosen to satisfy (B5), so $E(t) = 1$ is a consistency requirement between two exogenous processes, and nothing enforces it. If it fails the model does not break, the first k equations of each system continue to hold and only the last one does not, so sectoral equilibrium is compatible with macroeconomic disequilibrium.

If the condition were an identity the paper would have assumed its problem under investigation away; Proposition P1p shows why it is not. What is always true is that income per capita is spent, $\sum_i p_i c_i = e\, w$, at any level of employment, and with (C1) this gives $e = E$, which is (B6) below one. What does not follow, however, is that this has to be full employment income. In instances when this is not the case, the last equation of each system fails and the (un)employment is the residual. Pasinetti (1993) calls this an equilibrium of Keynesian underemployment and leaves it there; Section 7 takes it up.

So far the discussion about the condition was at a point in time. The question, however, is about its path. The following section differentiates E, and that is where the effects of AI adoption enter the picture.

4 AI as a shift in $\{\rho_i\}$: the Baumol inversion

In the previous section the condition was stated statically; this section follows its path through time. Differentiating E gives us the growth rate. Next we examine two situations: one which more or less corresponds to the stylized facts of the last two centuries, and one regime where AI raises productivity growth most where demand grows fastest and where this demand growth is being outpaced by the productivity increases.

4.1 The master equation

Proposition P2. *The master equation*

Differentiating (B4) and using (B3) and (C3),[3]

$$\frac{\dot{E}(t)}{E(t)} = \sum_i \mu_i(t)\,(r_i - \rho_i) \tag{P2}$$

The growth rate of E is a weighted mean of sectoral gaps between demand and productivity growth, where each is weighted by its expenditure share. Since $\sum_i \mu_i \rho_i$ is ρ^* (C5), it can be read as $\dot{E}/E = \sum_i \mu_i r_i - \rho^*$. The labour requirement remains steady when the demanded basket grows as fast as real income per capita, and it falls when real income outpaces demand. Labour released by growing productivity is re-employed only if demand grows to absorb it.

[3] $\dot{E} = \sum_i \dot{\varepsilon}_i = \sum_i \varepsilon_i (r_i - \rho_i)$ by (B3); dividing by E and writing $\mu_i = \varepsilon_i / E$ (C3) gives (P2).

With a common rate of productivity growth, ρ, a common rate of demand growth, r, (P2) gives $E(t) = E(0)\, e^{(r-\rho)t}$. This is Pasinetti's dynamic benchmark: full employment is sustained only if $r = \rho$ (Pasinetti 1993). The master equation (P2) is that condition, $r = \rho$, disaggregated by sector. The uniform case cannot show how the gaps are spread across sectors and which sectors carry the gap. This is what the rest of this section is about.

Demography has no direct effect on employment since population growth does not appear in the condition. It is stated per capita, so a shrinking or a growing population does not change E or its growth rate. Demography could act on employment through demand composition, for example: if an older population shifts demand towards care, whose labour coefficient is high, the r_i change, and E with them. This is not explored in the paper.

Even with the rates fixed, the weights, the expenditure shares μ_i, do not stay put. From (C3), μ_i grows at $(r_i - \rho_i) - \dot{E}/E$ and the weight moves towards sectors with the fastest growing labour requirement. As the weight moves, the growth rate of E rises: $\frac{d}{dt}(\dot{E}/E) = \sum_i \mu_i (r_i - \rho_i)^2 - (\dot{E}/E)^2 \geq 0$, the variance of the gaps across sectors, each weighted by its expenditure share. It is zero only when every sector has the same gap.

4.2 The phases device

$$\begin{aligned} c_i(t) &= c_i(\tau z)\, e^{r_i(\tau)\,\theta} \\ a_{ni}(t) &= a_{ni}(\tau z)\, e^{-\rho_i(\tau)\,\theta} \end{aligned} \qquad \theta = t - \tau z \in [0, z) \tag{B10}$$

If AI is seen as a disruption, the rates must be allowed to change. Pasinetti (1993) uses a device that divides time into phases of length z. The rates remain constant within a phase and (A2)–(A3) hold exactly. θ is the time elapsed since the phase began; at a boundary the rates change and the levels carry over (B10). The rest of the model remains the same. I use a single boundary where $\{\rho_i\}$ shifts while the r_i stay the same.

Since the levels carry over, E does not jump at the boundary; but its growth rate does. At the boundary, the expenditure shares are still at the pre-shock values, so the growth rate of E falls by the expenditure-weighted jump in the productivity growth, $\sum_i \mu_i \Delta\rho_i$, while ρ^* increases by the same amount. The shock is felt in proportion to the weight of the affected sectors and the pre-shock phase moves weight towards sectors with the largest positive gaps, which are the sectors where AI increases productivity growth the most - this is the premise of the paper. The weights are the largest in sectors hit with the largest shock.

4.3 Cost disease and its inversion

Proposition P3. *Baumol's cost disease is actually the stabiliser*

> E is stationary when the expenditure-weighted mean of $(r_i - \rho_i)$ is zero. Historically ρ_i and r_i have been negatively correlated across sectors — high-ρ agriculture and manufacturing with saturating demand (vis-à-vis productivity), so $r_i - \rho_i < 0$; and low-ρ services with high income elasticity, so $r_i - \rho_i > 0$. In the weighted sum the two offset: labour made redundant by technological progress was absorbed by the sectors with cost disease.

Baumol (1967) called the phenomenon a disease because the relative price of stagnant services could rise without end. Historically speaking, this process was one of the reasons why employment did not fall in spite of growing productivity in manufacturing (which outpaced the demand for manufacturing goods).[4] In the model, this stabiliser is merely an offset; no feedback or endogenous response is assumed. With constant rates, the drift of the weights would carry E ever further (away from one). Historically speaking, keeping E near one for two centuries meant the rates had to change and as demand saturated in some sectors, new sectors appeared (Proposition P11). That record is the foundation behind a common belief: that technological displacement is temporary, always offset in time by new industries that create employment.

Proposition P4. *The Baumol inversion and the no-reallocation result*

At the AI boundary, ρ_i rises most in the low-ρ, high-r sectors — precisely those that were the employment sink because of the cost disease — so the correlation sign flips. If $\rho_i > r_i$ for every i, then from (P2)

$$\min_i(r_i - \rho_i) \;\le\; \frac{\dot{E}}{E} \;\le\; \max_i(r_i - \rho_i) \;<\; 0 \tag{P4}$$

E declines monotonically. Reallocation of demand across existing k goods cannot restore it. At natural prices, a given budget commands the same labour whatever it buys (C1), and changing the weights (C3) cannot lift a weighted mean of negative gaps above zero.

The second part of Proposition P4 is the interesting one: shifting spending between goods does not help, because at natural prices a unit of spending employs the same labour whichever good it buys. More employment comes from more spending relative to the wage, but that requires the demand for at least one good to grow faster than its productivity. Under Proposition P4 this good does not exist. The resulting unemployment is structural in the strict sense: not transitional and not a matter of retraining.

The condition that $\rho_i > r_i$ in every sector is strong (it is, in effect, what the AGI/singularity scenario assumes), and it carries the result. It is enough that a single sector should keep $r_i > \rho_i$ and the drift of the weights would eventually turn the growth rate of E positive and E would recover. What would be the structural, political and social consequences of such a solution, where one sector employs nearly the whole labour force, is not part of this investigation and I leave it to the reader.[5] One way this happens is with human-in-the-loop floor of Section 6: where some labour cannot be automated by fiat and the measured productivity growth goes towards zero, and any sector with growing demand becomes such a sector. However, that case offers no reprieve: by Proposition P5 it hands the natural rate to whatever AI does worst.

[4] It was not the only one. Patnaik (2016) points to an outlet that a closed model leaves out: emigration to the temperate regions of white settlement. Citing W. Arthur Lewis, he puts each of the nineteenth century's two great migration streams, the European one among them, at "of the order of 50 million persons", and reports Utsa Patnaik's estimate that "almost half the number representing the increase in population each year in England between 1815 and 1910 migrated to the 'new world' ". On this reading, without that valve the technological unemployment discussed here might have become visible already in the nineteenth century.

[5] Nor need "eventually" be soon. With 60%, 39% and 1% of expenditure on gaps of -3%, -2% and $+1\%$ a year, E falls to about 7% of its starting level and turns after some 160 years. Which sector takes over the model does not say; on an étatist view, the state, best placed to keep its demand growing faster than its productivity.

Proposition P4 says that employment declines and nothing in the existing demand structure can stop it. It does not say where the economic system goes. There are three different scenarios that the paper investigates in this respect, and this is by no means an exhaustive list. Section 5 takes the labour coefficients to their extremes, reaching the boundary of the value system. Section 6 takes up the floor, the hours margin and new sectors. Section 7 asks what happens to the system when the condition fails and there is no rebound to a (socially desirable) level of E near one.

5 The boundary of the value system

The previous section dealt with the labour requirement as a whole; this section follows a single labour coefficient to its limit. The model we have can only describe a price system with goods that need some labour. $a_{ni} > 0$ is a condition of the value system (and not an incidental feature of it); what AI does is push the coefficients towards exactly the boundary. This can happen in three ways: it can approach but never reach zero, it can actually reach it or it can be held above it by a floor (either endogenous or institutional). This section investigates the first two; Section 6 tackles the third.

5.1 Asymptotic exit

Proposition P6. *Asymptotic exit is smooth*

> With ρ_j constant, $a_{nj} \to 0$ without reaching it. Then $p_j/p_i \to 0$ for every good i with $\rho_i < \rho_j$; $\varepsilon_j \to 0$ if $\rho_j > r_j$; and $\mu_j \to 0$ if some other sector has a larger gap $r_i - \rho_i$. Nothing in the system breaks; the sector quietly fades from the aggregates.

This is the normal case under (A2). A coefficient falling at a constant rate approaches zero but never quite reaches it. Importantly, the output isn't lost, it is just the sector's share that "leaves" the system. The good grows ever cheaper relative to goods whose productivity grows more slowly. Additionally, if its productivity outpaces its demand, the labour per capita it needs for production tends to zero. Likewise its expenditure share tends to zero too, as long as demand from some other sector outruns its productivity by more. This is the story of the industrial revolution: producible goods tend to get cheaper or better - the model only addresses the former. Its physical output keeps growing at $r_j + g$. The sector fades from aggregate value-based measure but its output keeps increasing (again, think farming, steel manufacturing etc.)

5.2 Exact extinction

$$a_{nj}(T) = 0 \text{ for finite } T \implies p_j = 0, \quad \mu_j = 0, \quad \varepsilon_j = 0 \tag{D0}$$

An extinction event is prevented under (A2) for any finite rate; the coefficient approaches but never reaches zero. Exact extinction is not a case within the model but its limit. Let the jump in ρ_j at the boundary grow without bound, and $a_{nj}(t)$ tends to zero at every moment after the boundary. That is a 'fully automated' sector and it is a limit of the model, not a new assumption.

Proposition P7. *Exact extinction: the free-gift theorem*

If $a_{nj} = 0$ at finite T then by (C1) $p_j = 0$ — the good is not cheap, it is *free*. Its labour requirement per capita and its expenditure share are zero, meaning its term drops out of the expenditure condition (C2) and out of (B5). The condition keeps its form over the remaining goods. At T, E falls by $\varepsilon_j(T)$. The labour from the sector is released and other sectors need to absorb it to keep full employment. The sector's labour productivity, $1/a_{nj}$, is undefined, so its "productivity growth" also loses its meaning. Wages per worker are w, the good doesn't add to measured income, real consumption includes X_j at price zero. In a pure-labour classical value theory, complete automation of a sector is formally identical to its product becoming a free gift of nature.

The dichotomy between value and use value is derived, not asserted. In spite of the rise in physical output, the contribution to income is zero. While odd to non-classicals, this is not a measurement error; zero is the right valuation for a good that is still produced but costs no labour. The issue is addressed further in Section 8. This withdrawal is also where the caveat of Section 3 bites. E before and after extinction at time T is computed over different sets of goods, and the fall of $\varepsilon_j(T)$ is exactly the redundant labour from the extinct sector.

5.3 Self-erasure

Proposition P5. *High productivity growth self-erases from the aggregate*

From (C3), $\mu_i(t) \propto \mu_i(0)e^{(r_i-\rho_i)t}$, so expenditure weight concentrates on $\operatorname{argmax}_i(r_i - \rho_i)$ and

$$\rho^*(t) \;\longrightarrow\; \rho_{i^*}, \qquad i^* = \operatorname*{argmax}_i (r_i - \rho_i) \tag{P5}$$

The asymptotic natural interest rate is the productivity growth rate of the sector with the fastest growing employment share.[6] Under the Proposition P4 inversion, this is the sector least automated by AI relative to its demand.

This is the result promised in Section 3 where it was noted that a sector's contribution to growth depends on how much is spent on it. The sectors where AI augments labour efficiency the most lose expenditure weight the fastest, because their falling prices outpace the growing demand for their output, meaning they account for a smaller share in ρ^*. Aggregate productivity growth converges on the productivity growth of the sector in which demand outruns productivity by the most, which under the inversion is the sector AI improves least relative to demand for it. The weight dominates the productivity growth of other sectors.[7] Extinction is this same mechanism pushed to its limit: a good whose labour coefficient reaches zero has a weight of zero. Its productivity growth is undefined and drops out of the aggregate altogether. In other words, high productivity growth erases itself from the aggregate. It is useful but not valued.

[6] Generically, one sector has the largest gap. If several share it, their weights keep their proportions at the start of the phase, and ρ^* converges on the mean of their ρ_i weighted by those proportions.

[7] Sector is selected based on the gap $r_i - \rho_i$ and not on productivity growth alone. Take two sectors with equal initial weights. In sector A, AI barely increases productivity ($\rho_A = 1\%$) but its demand does not grow ($r_A = 0$), so its gap is -1%. In sector B, AI raises productivity a lot ($\rho_B = 3\%$) but demand for its output grows faster still ($r_B = 5\%$), leaving a gap of $+2\%$. B's weight grows relative to A's at 3% a year: after a century B holds about 95% of expenditure and $\rho^* \approx 2.9\%$, converging on B's 3% rather than on A's 1%. A, the sector least touched by AI, is the one that drops out of the aggregate.

5.4 The model marks its own boundary

Plainly Proposition P7 does not describe a capitalist economy, where even full automation would not lead to a price falling to zero. This is a limitation of a pure-labour model, where machines appear only through the labour that builds and runs them, nothing is owned, everything that is produced is consumed, and no capital earns a return. A natural consequence of the system is that once the labour disappears so too does the price. This paper does not patch over this. With owned produced means of production that must earn a return, the automated good keeps a positive price even with labour coefficient falling to zero. This is the subject of the sequel. Within this paper, however, Section 6 takes the third route to the boundary: a floor below which the labour coefficient cannot fall, which keeps the axiom, and with it the value system, intact.

6 Two floors and two margins

Section 5 left the third route to the boundary for this section: a floor below which a labour coefficient cannot fall. The floor keeps the axiom of Section 3, and with it the value system, but it changes where the economy goes. This section takes the floor first, and then the two margins that might absorb the labour the floor does not: shorter hours and new goods.

6.1 The floor

$$\dot{a}_{ni} = -\rho_i \left(a_{ni} - \bar{a}_i\right), \qquad \bar{a}_i \geq 0 \tag{D1}$$

$$a_{ni}(t) = \bar{a}_i + \left(a_{ni}(0) - \bar{a}_i\right) e^{-\rho_i t} \tag{D1s}$$

$$\hat{\rho}_i(t) = -\frac{\dot{a}_{ni}}{a_{ni}} = \rho_i \left(1 - \frac{\bar{a}_i}{a_{ni}(t)}\right) \longrightarrow 0 \tag{D2}$$

$$\varepsilon_i(t) \sim \bar{a}_i c_i(t) \;\text{ for } \bar{a}_i > 0, \qquad E(\infty) = \sum_i \bar{a}_i \bar{c}_i \;\text{ under Engel saturation} \tag{D3}$$

The floor is a level $\bar{a}_i$ below which the labour coefficient of sector i cannot fall. It is the first added assumption in the paper. Productivity growth still proceeds at rate ρ_i, but only on the part of the coefficient above the floor (D1). Instead of the coefficient tending towards zero it decays towards $\bar{a}_i$ (D1s), and $\bar{a}_i = 0$ returns (A2). Measured productivity growth is the original rate scaled down by the share of the coefficient that can be automated (D2). It falls steadily towards zero because it runs out of things to automate. So an AI shock that raises ρ_i does not move the floor, it just speeds up the approach to it. Measured productivity growth jumps and then dies away, whereas without a floor it would jump and stay. If demand stops growing (a phase in which $r_i = 0$), total labour per head settles at a finite limit (D3).

6.2 The bifurcation

Proposition P8. *The floor bifurcation, in the two-disequilibria vocabulary*

> From (D1) and (D3), the floor sorts the long run into three cases, which fail the condition in two directions. In the first, with $\rho_i > r_i$ in every sector and no floor, output growth generates no employment and $E \to 0$: unemployment, unboundedly. In the second, with a floor and growing demand in any one sector ($\bar{a}_i > 0$ and $r_i > 0$ for some i), that sector's labour requirement grows with its output and $E \to \infty$: the economy needs more labour than it has, and since output cannot exceed what that labour can produce, the excess bears on prices rather than on quantities. With demand saturated in every sector ($r_i = 0$ from some phase on), $E \to \sum_i \bar{a}_i \bar{c}_i$, and full employment requiring that sum to equal one — a knife edge that generically fails, in either direction.

Proposition P8 sorts the long run into three cases (Figure 1). In the first with no floor and productivity outrunning demand everywhere, E falls towards zero: output grows but generates no employment. In the second, with a floor and growing demand, E may fall at first but turns round, because the floored sector's labour requirement cannot fall below $\bar{a}_i c_i$ and so grows with its output. This is the escaping sector of Section 4, and the case $E > 1$ where the economy needs more labour than it has. With a floor and demand that has stopped growing, E settles at a finite number and it holds only when that number is one.

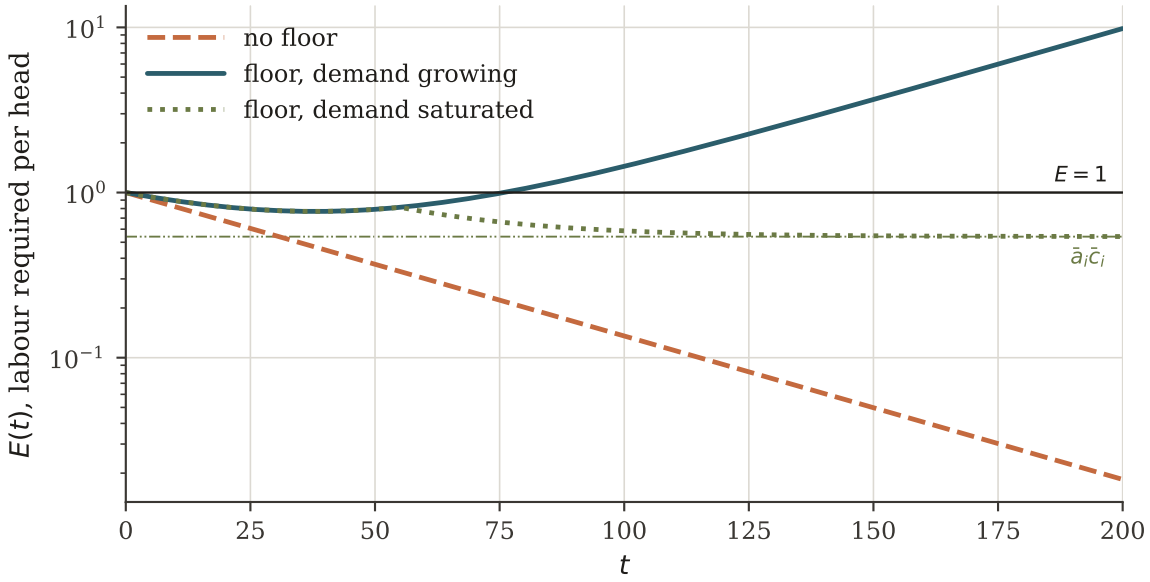


Figure 1: The three branches of Proposition P8, one sector each. The labour coefficient starts at one and falls at $\rho_i = 0.04$, towards a floor $\bar{a}_i = 0.18$ (no floor in the first branch); demand per head grows at $r_i = 0.02$, and in the third branch stops growing at $t = 55$, a phase boundary after which $r_i = 0$. Log scale. The values illustrate the mechanism and are not estimates.

The model has no money, so this excess demand is sustained upward pressure on the relative prices of floor-bound services, not a rate of inflation. Whether this would resolve itself in an actual economy, as rising prices, or some sort of rationing is an institutional question.

The second branch also rests on (A3): it needs demand for floor-bound goods to keep growing, while it follows from Proposition P9 that the floor stops the growth of real income per capita that drives demand. If demand follows income, the branch limits itself and the economy drifts into the third case.

6.3 Where the floor comes from

The floor has two sources. One is institutional: regulation, liability, auditability and compliance layers that require a person to answer for a decision. These pre-date AI and are part of modern industrial societies. While they might seem exogenous to a pure market-based view of an economic system (the natural state), I would argue, they are an endogenous albeit non-market response to the unemployment predicted by the model. That makes the floor a lever that governments, regulators and corporations can pull and Section 7 treats it as such.

The second is on the demand side. For some goods, being taught, served or cared for by a person is part of the good itself (the experience of it), as in the positional goods of Hirsch (1976), and goods valued because a person made or did them (think crafts, small farmers etc.). The floor is not a residue of failed automation but a desired feature of the good. This returns in Section 8 as the reason the economic problem is not simply solved.

6.4 The stationary state

Proposition P9. *Automation with a floor drives measured productivity growth to zero*

> If every sector has a floor, (D2) gives $\hat{\rho}_i(t) \to 0$ for all i, and (C5), evaluated at these measured rates, gives $\rho^*(t) \to 0$. The economy converges to a stationary state with measured growth and the natural rate of interest at zero, while physical output per head is at its historical maximum.

Proposition P9 is Proposition P5 with a floor. The natural rate of interest ρ^*, which is also the growth rate of real income per head, converges by Proposition P5 on the productivity growth of the sector with the largest expenditure weight, and under a floor, the growth tends to zero. The result is a classical stationary state, reached by automation instead of diminishing returns. It also explains why AI need not show up in productivity statistics: under a floor, measured productivity growth vanishes in the limit, even though output per head is at its peak.

6.5 The hours margin

$$E(t) = h(t) \qquad \Longrightarrow \qquad \frac{\dot{h}}{h} = \sum_i \mu_i (r_i - \rho_i) \tag{E1}$$

Proposition P10. *Shortening the working day*

> It follows from (E1) and Proposition P2 that $\dot{h}/h = \dot{E}/E$: the rate at which the working day must shorten to preserve full employment equals the expenditure-weighted excess of productivity growth over demand growth.

Pasinetti (1993, 49) already used shorter hours as one of the avenues for society to harvest the increased productivity. Instead of a quarter of the population out of work, everybody works three quarters of the time. Now with hours no longer fixed at one, full employment becomes $E = h$ (E1), the singularity condition (B8) again with demand per unit of labour $a_{in} = c_i/h$, so hours must change at the rate of E, and Proposition P10 turns that rate into a number. The shortened work week of Keynes (1930) becomes a quantity computed from the ρ_i and r_i. This avenue is applicable in one direction only: it can absorb a falling E, but it cannot supply a rising one, because hours (and working-age people) cannot grow without limit (at any given time). So it answers Proposition P4 and the first branch of Proposition P8, not the second.

6.6 The new-sector margin

$$R(t) = 1 - \sum_{i \le k} a_{ni}(t) c_i(t), \qquad \text{requiring} \qquad \sum_{j > k} a_{nj}(t) c_j(t) = R(t) \tag{E2}$$

Proposition P11. *New sector escape hatch needs its own labour coefficient*

> New goods only close the gap in (E2) through their labour coefficients. It is not enough they exist, their capacity as an escape hatch is bounded by their labour intensity — not by their number, usefulness or pace of arrival. A hundred new industries with zero labour coefficients absorb nobody.

New goods are the second way of taking the gains: as new goods to consume instead of more leisure time. (E2) defines $R(t)$, the share of labour that the existing k goods no longer absorb and which goods $k+1$ onward need to make up through their labour coefficients and per-capita demand. Proposition P11 says that what counts is how much labour a new good needs per head. And since a good's value here is its labour content, a new good born with a coefficient near zero is also born nearly free, at the boundary of Section 5. New goods stay exogenous, as Section 3 noted, and E after they enter is computed over a different set of goods. In Pasinetti (1993), learning does both: it raises productivity, which frees labour, and it creates new goods, which can absorb it. The learning that causes technological unemployment is also what can cure it. The question with AI and the looming singularity is whether or not these new goods require high enough labour coefficients to offset the initial decrease in employment.

Neither of these two avenues is guaranteed. The first, hours worked, can fall only so far (limited by the length of the working day), and new sectors only help if they employ people. If neither restores the condition, it fails and stays failed, and Section 7 asks what the economic system settles into when that happens.

7 The class-and-dole closure

When neither margin from Section 6 restores the condition, the economy is left where Pasinetti (1993) leaves it: in an equilibrium of Keynesian underemployment, where each sector satisfies its demand and only the condition for the system as a whole fails (Proposition P1p). He stops there, because the next step leaves the natural system and needs hypotheses about how agents behave within a given institutional set-up. This section takes half of that step. It specifies an institution but no behaviour, and derives by

accounting what employment must then be. The result is still a benchmark and not a forecast. What it shows is how technology and institutions together set employment, which keeps apart what Section 2 said the public debate runs together: what the technology permits and what the institutions do with it.

The paper has used one closure already without saying so. The employment rate of (B6), $e = E$, reads c_i as demand per head of the whole population, employed or not. The wage bill covers the consumption of the non-employed as well, and they consume exactly the same basket as the employed: the most egalitarian arrangement there is. It is the closure below with the partition switched off, (F4) with $A^u = A^e = E$ and $A^k = 0$. This partition lets the baskets differ.

7.1 The partition

$$A^e = \sum_i a_{ni} c_i^e, \qquad A^u = \sum_i a_{ni} c_i^u, \qquad A^k = \sum_i a_{ni} c_i^k \tag{F2}$$

$$e\,A^e + (1-e)\,A^u + A^k = e \tag{F3}$$

$$e = \frac{A^u + A^k}{1 + A^u - A^e} \tag{F4}$$

The partition is the paper's second added assumption, after the floor in (D1). The population is divided into the share of workers e, and the rest $1 - e$. Each has their own consumption basket. The elite can be either employed or non-employed; what they consume beyond either group's basket is a third basket, A^k, expressed per head of the whole population, not of the elite. (F2) measures the labour each basket absorbs: A^e for a worker's basket, A^u for the basket of someone out of work, A^k for the elite's. Nothing that produces is owned in a pure-labour economy, since there are no produced means of production, so the elite's claim on output does not come from property; here it is simply given, and ownership is the subject of the sequel. The reader can imagine courtiers or mandarins instead of capitalists in this group.[8] The labour market balances when the labour absorbed by the three baskets equals the labour the employed supply (F3), and solving for e gives employment as a function of the baskets (F4).

7.2 The underemployment equilibrium

Proposition P12. *The underemployment equilibrium*

Let a worker's own basket take less than their labour, $A^e < 1$. Then from (F4), $e \leq 1$ if and only if $A^e + A^k \leq 1$, with equality giving $e = 1$ exactly. Pasinetti's condition (B5) is the boundary case of the class system, recovered and not assumed. When $A^e + A^k < 1$, employment settles below unity and the non-employed are supported out of the surplus, at whatever level A^u sets. The system does not break, it reaches a different equilibrium.

[8] An elite's claim has also fed those who served it: retainers, soldiers, a court. The model needs no further class for them. Whatever the elite's claim supports independently of employment is part of A^k, and if those it supports provide a service the elite consumes, they are simply employed in that sector. Either way (F4) is unchanged, so a finer partition would rename its terms without adding a result.

Proposition P12 answers the question Proposition P1p left open: what the system does when the condition fails. Nobody is supported at full employment, so only the worker's basket and the elite's enter the condition; when the two together take exactly one unit of labour per head, $e = 1$, and that is (B5) again. Below that boundary, employment settles where (F4) puts it, the unemployed (and the elite) live on the surplus of the employed. Above the boundary, (F4) asks for more workers than are available: the excess-demand case of Section 6, in class form. The equilibrium does not stay put. As the labour coefficients fall, so do the labour contents of the baskets, and Proposition P14 says where employment goes.

7.3 Financing

$$e\left(1 - A^e\right) = (1 - e)\, A^u + A^k \tag{F5}$$

$$\frac{s}{v} = \frac{1 - A^e}{A^e} \tag{F6}$$

Proposition P13. *The dole is financed by construction; distribution is open*

> From (F5), the levy per employed worker is $1 - A^e$ — the fraction of their labour time not embodied in their own basket — and, summed over the employed, it exactly equals the labour absorbed by the dole and by elite consumption. The budget closes automatically. How the surplus is split between the non-employed and the elite is not determined.

Rearranged, (F3) reads as a budget (F5). Each employed worker supplies one unit of labour; A^e of it goes into their own basket, and the rest, $1 - A^e$, is a levy that provides for everyone who does not work. The budget closes because employment adjusts (and not because the levy is set right): a more generous dole requires more labour, so more people work and fewer need to be supported. In this model the dole has no financing problem; it has a distribution problem: how the surplus is split between those out of work and the elite. Nothing here says the workers accept the levy. That is a question of behaviour, which the closure leaves out, and leaving it out becomes more troublesome as automation proceeds: with baskets fixed, A^e falls, so an ever larger share of each worker's labour goes to those who do not work, and the rate of surplus value (F6) grows without bound.

7.4 Comparative statics

Proposition P14. *Comparative statics of the class system*

> All from (F4). Elite consumption always raises employment, $\partial e/\partial A^k = 1/(1 + A^u - A^e) > 0$. A more generous dole raises employment whenever there is underemployment, $\partial e/\partial A^u = (1 - A^e - A^k)/(1 + A^u - A^e)^2 > 0$. Workers producing for each other sustains employment, $\partial e/\partial A^e = (A^u + A^k)/(1 + A^u - A^e)^2 > 0$ whenever anyone works. And scaling every labour coefficient by $\lambda < 1$ with baskets fixed gives

$$e(\lambda) = \frac{\lambda(A^u + A^k)}{1 + \lambda(A^u - A^e)} \;\longrightarrow\; 0 \quad \text{as } \lambda \to 0 \tag{P14}$$

so universal productivity growth with fixed baskets drives employment to zero, and so does any growth of the baskets slower than that of productivity. Only baskets whose labour content does not decline sustain employment.

Proposition P14 collects what moves employment in (F4). Elite consumption always raises it, because producing that consumption takes workers, and so, below full employment, does a more generous dole. A worker's basket that takes more labour raises it too, for the mirror reason: each worker then has less surplus to spare, so it takes more of them to provide for everyone else. The last result is the one that matters for AI. Scale every labour coefficient down while the baskets stay fixed, and employment goes to zero (P14). This is Proposition P4 in class form: productivity growth alone empties the labour market, and only growing baskets hold employment up.

(F4) can also be read as a multiplier. Written as $e = (A^u + A^k)/\big(1 - (A^e - A^u)\big)$, its numerator is the labour absorbed by what would be consumed if nobody worked, the dole and the elite's consumption, and $A^e - A^u$ is what one more person in work adds to it, so the denominator is the part of a new worker's labour that their own consumption does not take back up, the leakage, as $1 - c$ is in Keynes's multiplier. With no dole and no elite, $e = 0$: if those who do not work consume nothing, nobody works, because a worker's own basket takes less than the labour they supply and nothing else buys the rest. This is the effective-demand reading of Proposition P1 again, now with classes in it.

7.5 Composition, not just level

The floor of Section 6 changes one thing. Coefficients then stop at $\bar{a}_i$ instead of falling to zero, and with baskets fixed employment settles at (F4) evaluated at the floors. That is above zero only if what the non-employed and the elite consume needs some floor-bound labour. What holds employment up is less the dole itself than what it consists of. Each basket in (F2) is a list of goods with a different labour content, so AI-produced bread and circuses, with near zero labour coefficients, absorb almost no labour, while care, teaching and entertainment delivered by people, the floor-bound goods of Section 6, absorb a great deal. This does not contradict Proposition P4. A budget of given value commands the same labour whatever it buys, but at natural prices a basket's cost in wage units is its labour content (C1): a dole of AI-produced goods is cheap because it employs nobody, and at the limit of Proposition P7 it is free. As with new goods (Proposition P11), what counts is the labour and not the usefulness. Graeber (2018)'s bullshit jobs in this setting are neither a moral failing nor an accident: they are part of the mechanism by which (F4) holds employment up, and if employment is to hold up as AI lowers the coefficients, the model says there must be more of them, not fewer.

The institutional floor of Section 6 is the same mechanism seen from the side of production. A rule that a person must sign off raises the labour coefficient of every good it touches, and with it all three baskets, each of which raises employment by Proposition P14. The governance layer creates both a floor on the coefficients and support that takes the form of a job.

7.6 A Roman precedent

The structure has a precedent in the classic account of the late Roman Republic, where conquest brought slaves in great numbers that displaced small farmers and many of them drifted to Rome. Rome's solution was more provision than re-employment: a public grain dole and public games, the *panem et circenses* of Juvenal's satire. Interpreted through (F4), the structure has three parts: an elite whose claim on output does not rest on its own labour (A^k), the displacement of free labour by something else — slaves then and AI now — and a dole that the displaced live on. In the model that dole is not charity outside the economy: somebody has to produce it, and in Rome that somebody was largely the provincial peasant (or slave) whose grain came as tribute.

The multiplier reading says why it is a necessity rather than an indulgence: with no dole and no elite consumption, nobody works. These are the unproductive consumers of Malthus (1820), doing the structural work on effective demand for which Keynes (1936) credited him. It is one of the less savoury adaptations of Section 1. It is not the technofeudalism of Varoufakis (2023), which concerns rent extracted through platforms; the analogy concerns only the structural position of a displaced population. Malthus and Keynes saw such consumption as what the system needs in order to work. It can also be read the other way: as the reason the system keeps the structure that makes it necessary. Section 8 returns to this.

8 Reading four public arguments

Section 1 named four public arguments about AI and promised to read them through the framework with its condition. Each reading below cashes out a result already derived, and none needs a new assumption. The aim is not to refute any of the claims. It is to see what (B5) says about each claim, and what each claim would need in order to be true.

8.1 A world without money

Elon Musk has argued that AI will make work optional and, most recently in an interview with the Economist, that money will not matter by 2036 (Musk 2026). In the pure-labour model this claim is true. Let AI adoption push every labour coefficient towards zero faster than demand grows, so that Proposition P4 holds in every sector and continues to hold. Every price goes to zero ((C1)) and, following Proposition P7, every good becomes a free gift of nature. This holds even in a class society. The labour absorbed by every basket goes to zero, and so do employment and the surplus, while the rate of surplus value grows without bound. The dole, the elite's consumption and the surplus become the same thing, nothing in value terms, because each is a quantity of labour. Nobody works (unless they want), nothing has a price, and labour content stops mattering because there is none. This is, in substance, the scenario he describes.

The reason it is true is the reason why it only holds inside the model. Prices in (C1) fall to zero because nothing in the model is owned: machines are in it only through the labour that builds and runs them, so technology is present (or measured, if you will) only in the labour coefficients it lowers, and no one earns a return on capital. Add capital goods that someone owns, as the sequel does, and a fully automated good still has a price,

$$p_j = p_K \, a_{Kj} \, (\delta + \pi_j) > 0 \tag{G1}$$

the value of capital good K needed per unit, $p_K a_{Kj}$, times depreciation δ plus the rate of profit π_j. Full automation does not, at least not in and of itself, abolish the price system. Labour is removed and ownership is left as the sole determinant of price and, unless other institutions step in, distribution in the natural system.

The claim is one of distribution wearing a technology claim's clothes. It also needs something the pure-labour model does not have: income that does not come from work. Here wages are the only claim on output, which is why Proposition P1 can treat full employment and the full spending of income as one condition. Once machines are owned, the question is who has a claim on what they produce, and this is a question for the same framework with capital goods, which is the subject of the sequel.

8.2 Solving the "economic problem"

Keynes (1930) expected the economic problem, the struggle for subsistence, to be solved within a century. The paper gives two reasons to doubt it. The first is Proposition P9: with a floor in every sector, measured productivity growth and the natural rate of interest both go to zero, so the economy arrives at a stationary state. This is not necessarily a state of abundance but it is the classical destination, reached from a different angle. Without a floor, abundance would arrive, and with it the world of the previous subsection.

The second, and probably stronger, point is the preference-side floor of Section 6, which denies the premise the previous subsection granted. Where being tutored, judged or served by a person is an integral part of the good, the coefficient stops at its floor.[9]

Demand for this type of positional goods does not saturate, because a position is defined relative to others (Hirsch 1976). Smith (1759, I.iii.2.1) wrote about it: what we pursue in "bettering our condition" is "to be observed, to be attended to, to be taken notice of with sympathy, complacency, and approbation". Human endeavour is just as much about ranking as about survival and material wellbeing. That part of the economic problem is zero-sum, and no productivity growth dissolves it. This is where both Keynes and the post-scarcity view err the same way: both hold the composition of wants fixed while their level is met (or seen as such). (A3) says the opposite, and the history of structural change is the evidence.

This consequence deserves its own paragraph. Positional demand is simultaneously the reason abundance does not arrive (by definition) and the mechanism that holds up employment in Proposition P14, since floor-bound goods are the labour-intensive baskets of Section 7. What keeps the economic problem alive is also what keeps people employed. Keynes (1930) saw the other side of this himself: once the economic problem was solved, man would be "...for the first time since his creation...faced with his real, his permanent problem—how to use his freedom from pressing economic cares, how to occupy the leisure, which science and compound interest will have won for him, to live wisely and agreeably and well". The model tells us where the problem would show up. As long as games of rank and being attended by other people are funnelled through the market, it is demand for floor-bound labour. If these activities move

[9] A case this paper leaves out: the AI revolution might change the perception of work itself. If it made human work a mark of distinction, demand for floor-bound goods would grow faster still. That is a shift in $\{r_i\}$, which this paper rules out by assumption; it would push the economy towards the second branch of Proposition P8.

outside the market, they still take up people's time, but not labour in the model's sense, and E no longer "sees" it.

8.3 The gains will not show up in the national income

Proposition P5 and Proposition P7 answer this directly. The sectors where AI (or any automation for that matter) contributes most to productivity growth contribute least to the natural rate of interest, because their falling prices outrun the growth of demand for them and their expenditure weight vanishes (think food in advanced economies). Aggregate productivity growth converges on that of the sector in which demand outruns productivity by most, under the inversion the one AI improves least relative to demand for it, and with a floor in every sector it goes to zero even as output per head peaks (Proposition P9).

In the limit the automated good has a price of zero and enters measured income at zero while its physical output keeps growing. The framework derives a wedge between value and use value that the measurement debate asserts. It is not a measurement error to be fixed. In a classical value theory, zero is the correct contribution to national income of a good that costs no labour: useful, as Section 5 put it, but not valued. The wedge is not a statistical artefact, though its prediction can be tested: sectors where productivity outgrows demand should see their share in the national income drop while their physical output grows. AI itself may be the first such sector: if the price of what it produces falls faster than demand for it grows, its share of spending shrinks even as its output multiplies.

8.4 Universal basic income solves it

In this model a basic income is a dole (Section 7): not a transfer outside the productive structure but a term in the condition that sets employment, (F4). It is financed by construction (Proposition P13), and below full employment a more generous one raises employment (Proposition P14), because somebody has to produce what it buys. Its recipients supply effective demand, as the unproductive consumers of Section 7 do, and narrow the gap to full employment without closing it (Proposition P12). How much it employs depends on what it buys. Fixed as a sum of money, it commands the same labour whatever it is spent on (Proposition P4); fixed as what it is meant to provide, one made of goods AI produces almost for nothing is cheap and, for that same reason, employs almost nobody, while one made of goods people provide, because a person is the point (sport) or because machines cannot yet do it (care), costs more and employs more.

Short of full automation, the dole is less a policy than a default: it is what the economy settles into when its institutions do not adapt to the fall in the labour it requires, or not fast enough, when hours are not shortened and no claim on what machines produce is created. Until every coefficient reaches zero and all goods become free, a basic income is made of goods that still need people, so it is paid for by those who work, and as automation proceeds an ever larger share of each worker's labour goes to those who do not ((F5), (F6)). The strain is on the road, not at its end: whether those who work will keep giving up that growing share is a question of behaviour, left out on purpose; even so, the result is a warning that the transition could break down long before abundance arrives. The technology lowers the labour required; whether the work that remains is carried by a minority, or the time it frees is shared out as leisure, depends on the institutions.

The alternatives are the model's other responses to a falling labour requirement. If the working day is shortened (Proposition P10), nobody supports anybody: there is no levy, and people still make for one another the human-made goods they want.[10] At the limit of full automation, what machines produce is free (Proposition P7) and a basic income becomes meaningless; with floors, set by rule, by what people want, or both, the economy stops short of that limit, either in the class economy or in the shared one; Pasinetti (1993, 56) saw both: "leisure time for all people or for some people". Whether floors are a loss depends on the road: with a well-managed transition, floors only delay the limit; however, if it is managed poorly, they are a safeguard that keeps work, and with it a claim on output, in place. The claim is usually meant differently, though: a basic income as everyone's share of automated production, a "universal high income" (Musk 2026), rather than a levy on wages. That needs income that does not come from work, and the pure-labour model has none: wages are its only claim on output, which is also why full employment is its benchmark. Who owns the machines' output, and whether a basic income can be paid from it, is the question of the sequel.

Read through one condition, each of the four claims rests on something it does not state. A world without money assumes that nothing is owned; a solved economic problem, that the composition of wants stays fixed and that people do not play zero-sum games (or games in general); gains "missing" from national income, that a gap between value and use value is an error; and a basic income, that a transfer only moves income around without changing who works. The first and the last come back to the same question, who owns what the machines produce, which a pure-labour model can name but not answer.

9 Conclusion and the road on

The paper has measured everything against one condition: that the labour needed per head, summed across sectors, equal one (B5) — which is at once the condition for full employment and for the full spending of income (Proposition P1). For most of two centuries of industrial development, the gaps between demand growth and productivity growth offset one another, because productivity grew slowest where demand grew fastest (Proposition P2, Proposition P3). If AI raises productivity growth most (and outpaces demand growth everywhere) in exactly these sectors, then the offset becomes a decline that no reallocation of demand can stop (Proposition P4). At the limit, a good that needs no labour is free, and sectors where AI augments productivity the most drop out of the aggregates (Proposition P7, Proposition P5). Regulations that already require human oversight of work AI could otherwise do alone may stop the economy short of that limit: they set a floor on labour coefficients where they apply (Section 6), and with a floor in every sector abundance becomes a stationary state (Proposition P8, Proposition P9). The other two options, shorter hours and new sectors, can absorb released labour, but nothing guarantees either (Proposition P10). Furthermore, the usual reassurance, that new sectors take up the slack as old ones are automated, holds only if the new sectors still need people (Proposition P11), and AGI, if it comes, is the case in which they may not.

When the condition fails, the economy does not break. It settles into an equilibrium in which some work and the rest are supported out of their surplus (Proposition P12). Financed by construction, this support leaves only distribution open (Proposition P13); employment then becomes a function of the labour

[10] This assumes that one person's hours can stand in for another's, as they can with homogeneous labour (Section 2). In a highly specialised economy they often cannot; how work would be shared there is beyond this paper.

contained in the baskets of those who do not work (Proposition P14). Importantly, the same technology is compatible with different societies: one in which a levy on a shrinking workforce supports everyone else, an elite included, and one in which what machines make is free and people work a few hours for one another. The model cannot answer which one emerges. It says what each requires, and that is the gap between what the technology permits and what institutions deliver that Section 2 set out to measure.

The model marks its own limits. Machines enter it through the labour needed to build and run them, and they earn no return, so a good that needs no labour, not even indirectly in its machines, is free (Proposition P7). Once capital goods are owned, the good keeps a price, namely depreciation plus profit (G1), and ownership, rather than labour, decides who receives it; at the limit, ownership alone keeps the price above zero. That is the task of the sequel. Adding capital to the framework resolves the extinction scenario identified in Section 5 and makes three questions answerable: who owns machine-generated wealth, whether a basic income can be funded directly from it rather than levied on remaining workers, and what leverage people retain when much of their labour is no longer needed. Two extensions are left open. With labour of more than one kind (Section 2), unemployment could become structural well before AI outmatches people at every task, once it outmatches them where many are employed whose skills decay out of work and are of little use elsewhere. With more than one country, a country can keep its people employed by exporting, moving the shortfall onto those who import.

The singularity of the title is not the moment at which machines outrun us. It is a vanishing determinant, the condition under which an economy needs exactly the labour it has (B8). AI, above all if it reaches the AGI singularity, can move an economy away from this condition, perhaps very swiftly, but it does not decide where the economy goes. That is left to the choices this condition forces, and Pasinetti (1993, 57) explains why they cannot be avoided:

> It imposes, at every moment in time, not just the possibility, but the necessity of making choices. There does not exist the alternative of not choosing! Technical progress does not present itself in the form of a free gift [...]. Rather it presents itself in the form of an impetuous flow which cannot be stopped, but which must always be channelled into new directions, which must themselves, bit by bit, be discovered and invented, as the old ones continuously become saturated. And if the new directions do not emerge, or are not found, or are not found soon enough, that impetuous flow [...] may well [...] cause considerable damage (in the form of unemployment) to existing positions.

Complete automation in the pure-labour model makes goods formally free gifts of nature, but the progress that makes them is not a free gift. Public debate treats a disagreement about institutions as a disagreement about the technology (Section 2); this paper has used a framework that keeps the two apart, and read four of the more popular current claims through it. Whether output so produced actually becomes free is a matter of institutional adaptation and not just adoption. The road matters as much as the end: on the way to free goods, a class-and-dole economy asks those still at work for an ever larger share of their time, the levy of Proposition P13 tending to the whole of it, while shorter hours need no levy but must fall as fast as employment does (Proposition P10). This investigation did not rank these alternatives, but the extremes some of them reach are a comment on their political viability and desirability.

The stakes are not only political. A more regulated world of institutional floors rewards credentials, a

culture closer to a mandarinate, whereas a world of free goods moves the contest for rank outside of the market mechanism. Neither is developed further, but these choices, that will inevitably have to be made, will shape more than just the economy: they will shape our ethos, our priorities, our ideals and how we live with one another.

10 Appendix: the equation set

The spine of the paper, in the order the equations appear. This table is extracted from the text at build time — there is no second copy to keep in step.

(A1) $N(t) = N(0)\, e^{gt}$

(A2) $a_{ni}(t) = a_{ni}(0)\, e^{-\rho_i t}$

(A3) $c_i(t) = c_i(0)\, e^{r_i t}$

(B1) $X_i(t) = c_i(t)\, N(t) = X_i(0)\, e^{(r_i+g)t}$

(B2) $L_i(t) = a_{ni}(t)\, X_i(t)$

(B3) $\varepsilon_i(t) = \frac{L_i(t)}{N(t)} = a_{ni}(t)\, c_i(t) = \varepsilon_i(0)\, e^{(r_i-\rho_i)t}$

(B4) $E(t) \;\equiv\; \sum_i a_{ni}(t)\, c_i(t)$

(B5) $E(t) = \sum_i a_{ni}(t)\, c_i(t) = 1$

(B6) $e(t) = \min\{E(t),\, 1\}$

(B7a) $$X_i - a_{in} X_n = 0, \qquad i = 1, \ldots, k$$
$$X_n - \sum_i a_{ni} X_i = 0$$

(B7) $$A = \begin{bmatrix} 1 & 0 & \cdots & 0 & -a_{1n} \\ 0 & 1 & \cdots & 0 & -a_{2n} \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & 1 & -a_{kn} \\ -a_{n1} & -a_{n2} & \cdots & -a_{nk} & 1 \end{bmatrix}$$

(B8) $\det A = 1 - \sum_i a_{ni} a_{in}$

(B9) $A^{\top} p = 0$

(C1) $p_i(t) = a_{ni}(t)\, w(t)$

(C2) $\sum_i p_i(t)\, c_i(t) = w(t)$

(C3) $\mu_i(t) = \frac{p_i c_i}{\sum_j p_j c_j} = \frac{a_{ni} c_i}{E(t)}$

(C4) $\mu_i(t) = \varepsilon_i(t)$

(C5) $\rho^*(t) = \sum_i \mu_i(t)\, \rho_i$

(C6) $\frac{w}{p_i} = \frac{1}{a_{ni}}$

(P2) $\frac{\dot{E}(t)}{E(t)} = \sum_i \mu_i(t)\, (r_i - \rho_i)$

(B10) $$c_i(t) = c_i(\tau z)\, e^{r_i(\tau)\,\theta}$$
$$a_{ni}(t) = a_{ni}(\tau z)\, e^{-\rho_i(\tau)\,\theta} \qquad \theta = t - \tau z \in [0, z)$$

(P4) $\min_i(r_i - \rho_i) \;\le\; \frac{\dot{E}}{E} \;\le\; \max_i(r_i - \rho_i) \;<\; 0$

(D0) $a_{nj}(T) = 0$ for finite $T \implies p_j = 0, \quad \mu_j = 0, \quad \varepsilon_j = 0$

(P5)	$\rho^*(t) \longrightarrow \rho_{i^*}, \qquad i^* = \text{argmax}_i\,(r_i - \rho_i)$
(D1)	$\dot{a}_{ni} = -\rho_i\,\big(a_{ni} - \bar{a}_i\big), \qquad \bar{a}_i \geq 0$
(D1s)	$a_{ni}(t) = \bar{a}_i + \big(a_{ni}(0) - \bar{a}_i\big)e^{-\rho_i t}$
(D2)	$\hat{\rho}_i(t) = -\frac{\dot{a}_{ni}}{a_{ni}} = \rho_i\left(1 - \frac{\bar{a}_i}{a_{ni}(t)}\right) \longrightarrow 0$
(D3)	$\varepsilon_i(t) \sim \bar{a}_i c_i(t)$ for $\bar{a}_i > 0, \qquad E(\infty) = \sum_i \bar{a}_i \bar{c}_i$ under Engel saturation
(E1)	$E(t) = h(t) \qquad \Longrightarrow \qquad \frac{\dot{h}}{h} = \sum_i \mu_i (r_i - \rho_i)$
(E2)	$R(t) = 1 - \sum_{i \leq k} a_{ni}(t) c_i(t), \qquad$ requiring $\qquad \sum_{j>k} a_{nj}(t) c_j(t) = R(t)$
(F2)	$A^e = \sum_i a_{ni} c_i^e, \qquad A^u = \sum_i a_{ni} c_i^u, \qquad A^k = \sum_i a_{ni} c_i^k$
(F3)	$e\,A^e + (1-e)\,A^u + A^k = e$
(F4)	$e = \frac{A^u + A^k}{1 + A^u - A^e}$
(F5)	$e\left(1 - A^e\right) = (1-e)\,A^u + A^k$
(F6)	$\frac{s}{v} = \frac{1 - A^e}{A^e}$
(P14)	$e(\lambda) = \frac{\lambda(A^u + A^k)}{1 + \lambda(A^u - A^e)} \longrightarrow 0 \quad \text{as } \lambda \to 0$
(G1)	$p_j = p_K\,a_{Kj}\,(\delta + \pi_j) > 0$

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